

CORRELATION OF  $^{13}\text{C}$ -H SPIN-SPIN COUPLING CONSTANTS  
AND INTERNUCLEAR ANGLES IN CYCLIC MOLECULES

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(Received 17 April 1964)

A linear function correlates the  $^{13}\text{C}$ -H nuclear spin-spin coupling constant  $J$  (in c.p.s.) and the fractional s-character ( $\rho_{\text{H}}$ ) of the carbon a.o. from which the C-H bond is formed<sup>1</sup>:

$$J = 500 \rho_{\text{H}} \quad \underline{1}$$

Recently Foote<sup>2</sup> has reported that  $J$  in cycloalkanes appears to be related to the internuclear C-C-C bond angle  $\theta_{\text{n}}$  by a linear function. Similarly, Laszlo and Schleyer<sup>3</sup> have observed a linear relationship between  $J$  and the C-C-C internuclear bond angle in C-CH=C systems.

Foote's data are reasonably approximated (Table I) by equation 2:

$$J = 203 - 0.720 \theta_{\text{n}} \quad \underline{2}$$

and from 1 and 2 it follows that  $\theta_{\text{n}}$  is a linear function of  $\rho_{\text{H}}$ . This result is surprising since "hybridization would be expected to be more closely connected with interorbital than interatomic angle"<sup>2</sup>.

Using normalized and orthogonalized hybrid bonding orbitals<sup>4</sup>, the following relationships obtain for the average local  $C_{2v}$  symmetry of the central carbon atom in  $\text{C}-\text{CH}_2-\text{C}$ :

$$2\rho_{\text{H}} + 2\rho_{\text{C}} = 1 \quad \underline{3}$$

$$\rho_{\text{C}} (1 + \lambda^2) = 1 \quad \underline{4}$$

$$\lambda^2 \cos \theta_0 = -1 \quad \underline{5}$$

where  $\rho_{\text{C}}$  is the fractional s-character of the central carbon a.o. from which the C-C bond is formed,  $\lambda$  is the mixing coefficient which determines the p-character of the hybrid  $(s+\lambda p)$  a.o. bonding to carbon, and  $\theta_0$  is the interorbital C-C-C bond angle, i.e. the angle subtended by the orbital axes at the central carbon atom. Combining 1, 3, 4 and 5:

$$J = \frac{250 (1 + \cos \theta_0)}{(1 - \cos \theta_0)} \quad \underline{6}$$

When  $\theta_n$  is taken as equal to  $\theta_0$ , it is found (Table I) that equation 6 does not satisfactorily correlate  $J$  and  $\theta_n$  except for the case of perfect tetrahedral symmetry ( $\theta_n = 109.5^\circ$ ,  $J = 125$  c.p.s.). This is the position of minimum angle strain and of maximum overlap where the bonding orbital axes and the internuclear lines coincide, i.e.  $\theta_n = \theta_0$  at  $109.5^\circ$ . It has been pointed out<sup>4,5</sup> that any departure from this angle which is introduced by the geometric constraints of ring formation destroys the  $\sigma$ -character of the bond and results in bond "bending".

Let us now postulate (a) that with increasing angle bending,

expressed as  $\Delta\theta_n = |\theta_n - 109.5^\circ|$ , there is a corresponding increase in orbital bending  $\Delta\theta_o = |\theta_o - 109.5^\circ|$ , where  $\Delta\theta_o < \Delta\theta_n$  for a given  $\theta_n$  (orbital following), and (b) that the correspondence between the two changes may be expressed to a first approximation by a linear relationship, i.e., that  $d\Delta\theta_o/d\Delta\theta_n$  is a constant,  $c$  ( $c \neq 1$ ). Integration gives:

$$\theta_o = c\theta_n + 109.5(1-c) \quad 7$$

The constant  $c$  may be evaluated by fixing  $\theta_o$  at  $102^\circ 52'$  for  $\theta_n = 60^\circ$ , the value for cyclopropane<sup>6</sup>, whence:

$$\theta_o = 0.1340 \theta_n + 94.83 \quad 8$$

Combination of 6 and 8 gives  $J$  as a non-linear function of  $\theta_n$ . Agreement with found values is good (Table I). Even though the analytical expression is not linear, the curvature of the plot of  $J$  vs.  $\theta_n$  is so slight that the combined function closely approximates Foote's empirical straight line<sup>2</sup>.

The present approach may be extended to methine systems  $\text{R}_3\text{CH}$  having local  $\text{C}_{3v}$  symmetry at the carbon atom bearing the hydrogen. For normalized and orthogonalized orbitals:

$$\rho'_H + 3\rho'_C = 1 \quad 9$$

For a given value of  $\theta_n$  (and thus of  $\theta_o$ ) in the local  $\text{C}_{2v}$  and  $\text{C}_{3v}$  systems,  $\rho_C = \rho'_C$ . Therefore, combining 1, 3 and 9:

$$J' = 3J - 250 \quad 10$$

where  $J'$  is the  $J_{\text{C-H}}$  spin-spin coupling constant of the local  $\text{C}_{3v}$  system. The empirical correlation 2 for  $\text{C}_{2v}$  thus becomes 11 for  $\text{C}_{3v}$ , and the

analytical expression 6 is converted to 12:

$$J' = 359 - 2.16 \theta_n \quad \underline{11}$$

$$J' = \frac{500 (1 + 2 \cos \theta_0)}{(1 - \cos \theta_0)} \quad \underline{12}$$

Combination of 12 and 8 then affords  $J'$  as a function of  $\theta_n$ . This treatment gives a  $J$ -value for the cubane system which is in satisfactory agreement with the experimental quantity<sup>7</sup> (Table II).

With the appropriate modification<sup>4</sup> of equations 3 - 5 and with a knowledge of the molecular geometry, the present treatment might be extended to systems of lower symmetry than local  $C_{nv}$ . For example, in bicyclo[2.2.1] heptadiene, where the bond angles are known<sup>8</sup>, the bridge-head carbon has local  $C_s$  symmetry and we calculate a  $J_{\text{C-H}}$  of 145.6 c.p.s. This is in good agreement with the found<sup>9</sup> value of 146 c.p.s.

An extension of the preceding arguments leads to several predictions:

(a) In bicyclo[1.1.1]pentane<sup>10</sup> the  $\theta_n$ -values of the methine ( $\theta_1$ , local  $C_{3v}$  symmetry) and methylene ( $\theta_2$ , local  $C_{2v}$  symmetry) carbons are correlated by the geometric relationship  $2(1 - \cos \theta_1) = 3 \cos^2 \theta_2 / 2$ . It follows that the two values of  $J_{\text{C-H}}$  should be linked together by equations 2 and 11, or by equations 6, 12 and 8.

(b) Tetrahedrane ( $C_4H_4$ ), which has local  $C_{3v}$  symmetry, should have a  $J_{\text{C-H}}$  of about 225 c.p.s. (Table II), i.e. lower than that of acetylene<sup>1</sup> (251 c.p.s.) but higher than those of tricyclo[4.1.0.0<sup>2,7</sup>]heptane<sup>11</sup> (200 c.p.s.) and 4,5-diphenyltricyclo[1.1.1.0<sup>4,5</sup>]pentan-2-one<sup>12</sup> (190 c.p.s.).

(c) Prismane or "Ladenburg benzene" ( $C_6H_6$ ), which has local  $C_s$  symmetry,

should have a  $J_{\text{C-H}}$  of about 180 c.p.s.

We note that relationships not unlike those proposed in this paper may also obtain in the ground state triplet methylenes<sup>13</sup>. E.s.r. measurements ( $^{13}\text{C}$  hyperfine splittings) suggest that bond bending in fluorenylidene involves orbital following similar in magnitude to that given by our equation 8.

Table I

Compound	$\theta_n^a$	Found <sup>a</sup>	$J_{\text{C-H}}$ (c.p.s.)		
			[1]	[2]	[3]
Cyclopropane	60.0	161	159.8	750.1	158.9
Cyclobutane	89.3	134	138.7	256.2	138.0
Cyclobutanone	90.0	133.5	138.2	250.0	137.5
Norbornadiene-7	96.7	135.5	133.4	197.7	133.1
Cyclopentane	103.3	128	128.6	156.5	128.8
Cyclohexane	110	123, 124	123.8	122.6	124.6
Cycloheptane	112	123	122.4	113.7	123.3
Cyclooctane	112	122	122.4	113.7	123.3
Cyclodecane	116	118	119.5	97.6	121.0
Cyclododecane	112	123	122.4	113.7	123.3

<sup>a</sup> Data from references (1) and (2).

[1] Calcd. from equation 2.

[2] Calcd. from equation 6 assuming  $\theta_o = \theta_n$

[3] Calcd. from equations 6 and 8.

Table II

Compound	$\theta_n$	Found	[1]	[2]
Tetrahydrofuran	60.0	?	229.4	226.7
Cubane	90.0	$160 \pm 5^a$	164.6	162.5
Adamantane	109.5	(125)	122.4	125.0

<sup>a</sup>Based on dicarbomethoxycubane, ref. 7.

[1] Calcd. from equation 11

[2] Calcd. from equations 12 and 8

#### Acknowledgments

I am grateful to several of my colleagues for stimulating discussions and helpful suggestions: Gerhard L. Closs (Chicago), Marshall Gates (Rochester), Albert Moscowitz (Minnesota), Paul von R. Schleyer (Princeton), Edel Wasserman (Bell Telephone Labs.) and Kenneth B. Wiberg (Yale).

#### REFERENCES

- (1) N. Muller and D. E. Pritchard, *J. Chem. Phys.*, 31, 768, 1471 (1959); J. N. Shoolery, *ibid.*, 31, 1427 (1959); N. Muller, *ibid.*, 36, 359 (1962); C. Juan and H. S. Gutowsky, *ibid.*, 37, 2198 (1962); E. Lippert and H. Prigge, *Ber. Bunsen Ges. Physik. Chem.*, 67, 415 (1963).
- (2) C. S. Foote, *Tetrahedron Letters*, No. 9, 579 (1963).
- (3) P. Laszlo and P. von R. Schleyer, *J. Am. Chem. Soc.*, 86, 1171 (1964).
- (4) C. A. Coulson, "Valence", Oxford University Press, 1952, Ch. 3
- (5) C. A. Coulson and W. E. Moffitt, *Phil. Mag.*, 40, 1 (1949).
- (6) C. A. Coulson and T. H. Goodwin, *J. Chem. Soc.*, 3161 (1963).
- (7) P. E. Eaton and T. W. Cole, Jr., *J. Am. Chem. Soc.*, 86, 962 (1964).
- (8) V. Schomaker, cited by C. F. Wilcox, Jr., S. Winstein and W. G. McMillan, *J. Am. Chem. Soc.*, 82, 5450 (1960).
- (9) K. Tori, R. Muneyuki and H. Tanida, *Canad. J. Chem.*, 41, 3142 (1963).
- (10) K. B. Wiberg, D. S. Connor and G. M. Lampman, *Tetrahedron Letters*, No. 10, 531 (1964).
- (11) G. L. Closs and L. E. Closs, *J. Am. Chem. Soc.*, 85, 2023 (1963).
- (12) S. Masamune, *J. Am. Chem. Soc.*, 86, 735 (1964).
- (13) E. Wasserman, private communication.